

Decoupling and antiresonance in a quantum dot chain with two neighboring dots coupled to both leads

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Electron transport through a quantum dot chain with two neighboring dots coupled to both leads is theoretically studied. In such a system, it is found that only for the even-numbered quantum dot structure with the same-number quantum dots coupled to each connecting dot, some eigenstates of the quantum dots decouple from the leads. Namely, all odd eigenstates decouple from the leads in the absence of magnetic flux, but all even eigenstates will decouple from the leads when a magnetic flux is introduced. In addition, by adjusting the magnetic fluxes through any subring, some eigenstates decouple from one lead but still couple to the other, and then some new antiresonances occur.

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I. INTRODUCTION

In the past years, electronic transport through quantum-dot(QD) systems has been extensively studied both experimentally and theoretically. The atom-like characteristics of a QD, such as the discrete electron levels and strong electron correlation, manifest themselves by the experimental observations of Coulomb blockade[1, 2, 3, 4], conductance oscillation[5], and Kondo effect[6, 7, 8, 9] in the electronic transport process through a QD. Therefore, a single QD is usually called an artificial atom, and a mutually coupled multi-QD system can be regarded as an artificial molecule. With the progress of nanotechnology, it now becomes possible to fabricate a variety of coupled QD structures with sizes to be smaller than the electron coherence[10, 11]. Thereby, more and more recent experimental investigations focus on the electronic transport property through coupled QD systems[12, 13, 14, 15, 16]. In comparison with a single QD, coupled QD systems possess higher freedom in implementing some functions of quantum devices, such as the QD cellular automata[17] and solid-state quantum computation[18, 19].

Many experimental and theoretical works have become increasingly concerned about the electronic transport through various multi-QD systems[20, 21, 22, 23, 24, 25, 26, 27]. According to the previous research, we know that characteristic of the linear conductance spectrum consists in the conductance peaks reflecting the eigenlevels of the coupled QDs. Moreover, the zero point of the conductance, called antiresonance, has also received much attention, which is interpreted as the destructive quantum interference among electron waves going through different paths. Typically, in the structures with one or several quantum dots side-coupled to a main conducting channel, the antiresonant points coincide with the eigenenergies of the dangling QDs. Such a theoretical prediction about antiresonance has been observed experimentally[20, 21], which stimulated further theoretical interest in this topic[23, 24, 25, 26, 27]. Based on the properties of antiresonance, the applications of some QD structure are proposed[27, 28, 29].

However, for several QD systems[30, 31], not all the eigenlevels appear in the conductance spectrum and the corresponding eigenlevels are completely localized, which originate from these quantum states decoupled from the leads. Thereby, the occurrence of decoupling modifies the quantum interference of the corresponding structure in the nontrivial way, so that the electron transport properties are changed. For instance, it can also give rise to the appearance of antiresonance. Accordingly, such topics have become a direction of focusing on the electron transport through the coupled QDs. Motivated by these previous works, we pay attention to the electron transport through a quantum dot chain, in which both the leads couple to two neighboring dots of the chain. As a result, only for the even-dot structure with the same-number quantum dots coupling to the connecting dots, some of the eigenstates of the coupled QDs decouple from the leads. Namely, in the absence of magnetic flux only the odd eigenstates decouple from the leads, while the even eigenstates will decouple from the leads when magnetic flux is introduced. But the antiresonance in the conductance

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spectrum is irrelevant to the tuning of magnetic flux. In addition, by adjusting the magnetic fluxes through each subring, one can achieve the decoupling of some eigenstates from one lead but still coupled to the other, which causes the occurrence of new antiresonance.

II. MODEL

The coupled-QD structure we consider is illustrated in Fig.1(a). The Hamiltonian to describe the electronic motion in such a structure reads

$$H = H_C + H_D + H_T. \quad (1)$$

The first term is the Hamiltonian for the noninteracting electrons in the two leads:

$$H_C = \sum_{k\alpha \in L,R} \varepsilon_{k\alpha} c_{k\alpha\sigma}^\dagger c_{k\alpha\sigma}, \quad (2)$$

where $c_{k\alpha\sigma}^\dagger$ ($c_{k\alpha\sigma}$) is an operator to create (annihilate) an electron of the continuous state $|k, \sigma\rangle$ in lead- α with σ being the spin index, and $\varepsilon_{k\alpha}$ is the corresponding single-particle energy. The second term describes electron in the QD chain. It takes a form as

$$H_D = \sum_{m=1}^N \varepsilon_m d_{m\sigma}^\dagger d_{m\sigma} + \sum_{m=1}^{N-1} (t_m d_{m+1\sigma}^\dagger d_{m\sigma} + \text{H.c.}), \quad (3)$$

where $d_{m\sigma}^\dagger$ ($d_{m\sigma}$) is the creation (annihilation) operator of electron in QD- m , ε_j denotes the electron level in the corresponding QD. We assume that only one level is relevant in each QD and the value of ε_m is independent of $\varepsilon_m = \varepsilon_0$. When the sequence numbers of the two neighboring dots coupled to both leads are taken as j and $j+1$, the last term in the Hamiltonian describes the electron tunneling between the leads and QDs. It is given by

$$H_T = \sum_{k\alpha\sigma} (V_{\alpha j} d_{j\sigma}^\dagger c_{k\alpha\sigma} + V_{\alpha j+1} d_{j+1\sigma}^\dagger c_{k\alpha\sigma} + \text{H.c.}), \quad (4)$$

where $V_{\alpha j}$ and $V_{\alpha j+1}$ with $\alpha = L, R$ denotes the QD-lead coupling strength. We adopt a symmetric QD-lead coupling configuration which gives that $V_{Lj} = V e^{i\phi_L/2}$, $V_{Lj+1} = V e^{-i\phi_L/2}$, $V_{Rj} = V e^{-i\phi_R/2}$, and $V_{Rj+1} = V e^{i\phi_R/2}$ with V being the dot-lead coupling strength. The phase shift ϕ_α is associated with the magnetic flux Φ_α threading the system by a relation $\phi_\alpha = 2\pi\Phi_\alpha/\Phi_0$, in which $\Phi_0 = h/e$ is the flux quantum.

To study the electronic transport properties, the linear conductance of the noninteracting system at zero temperature is obtained by the Landauer-Büttiker formula

$$\mathcal{G} = \frac{e^2}{h} \sum_{\sigma} T_{\sigma}(\omega)|_{\omega=\varepsilon_F}, \quad (5)$$

$T(\omega)$ is the transmission function, in terms of Green function which takes the form as [32, 33]

$$T_{\sigma}(\omega) = \text{Tr}[\Gamma^L G_{\sigma}^r(\omega) \Gamma^R G_{\sigma}^a(\omega)], \quad (6)$$

where Γ^L is a $N \times N$ matrix, describing the coupling strength between the two connecting QDs and the left lead. It is defined as $[\Gamma^L]_{mm'} = 2\pi V_{Lm} V_{Lm'}^* \rho_L(\omega)$. We will ignore the ω -dependence of $\Gamma_{mm'}^L$ since the electron density of states in the left lead, $\rho_L(\omega)$, can be usually viewed as a constant. By the same token, we can define $[\Gamma^R]_{mm'}$. In fact, one can readily show that $[\Gamma^L]_{mm} = [\Gamma^R]_{mm}$ in the case of identical QD-lead coupling. Hence we take $\Gamma = [\Gamma^L]_{mm} = [\Gamma^R]_{mm}$ to denote the QD-lead coupling function. In Eq. (6) the retarded and advanced Green functions in Fourier space are involved. They are defined as follows: $G_{mm',\sigma}^r(t) = -i\theta(t)\langle\{d_{m\sigma}(t), d_{m'\sigma}^\dagger\}\rangle$ and $G_{mm',\sigma}^a(t) = i\theta(-t)\langle\{d_{m\sigma}(t), d_{m'\sigma}^\dagger\}\rangle$, where $\theta(x)$ is the step function. The Fourier transforms of the Green functions can be performed via $G_{mm',\sigma}^{r(a)}(\omega) = \int_{-\infty}^{\infty} G_{mm',\sigma}^{r(a)}(t) e^{i\omega t} dt$. These Green functions can be solved by means of the equation-of-motion method. By a straightforward derivation, we obtain the retarded Green functions which are written in a matrix form as

$$G_{\sigma}^r(\omega) = \begin{bmatrix} \ddots & & & & & & \\ -t_{j-2} & g_{j-1\sigma}(z)^{-1} & -t_{j-1} & 0 & 0 & 0 \\ 0 & -t_{j-1} & g_{j\sigma}(z)^{-1} & -t_j + i\Gamma_{j,j+1} & 0 & 0 \\ 0 & 0 & -t_j + i\Gamma_{j+1,j} & g_{j+1\sigma}(z)^{-1} & -t_{j+1} & 0 \\ 0 & 0 & 0 & -t_{j+1} & g_{j+2\sigma}(z)^{-1} & -t_{j+2} \\ \vdots & & & & & \ddots \end{bmatrix}^{-1}, \quad (7)$$

with $z = \omega + i0^+$, $g_{m\sigma}(z) = (z - \varepsilon_m + i\Gamma_{mm})^{-1}$, being the zero-order Green function of the QD- m unperturbed by another QD, and $\Gamma_{mm'} = \frac{1}{2}([\Gamma^L]_{mm'} + [\Gamma^R]_{mm'})$. In addition, the advanced Green function can be readily obtained via a relation $G_\sigma^a(\omega) = [G_\sigma^r(\omega)]^\dagger$.

Notice that the linear conductance spectrum of the coupled QD structure reflects the eigenenergy spectrum of the “molecule” made up of the coupled QDs. In other words, each resonant peak in the conductance spectrum represents an eigenenergy of the total QD molecule, rather than the levels of the individual QDs. Therefore, it is necessary to transform the Hamiltonian into the molecular orbital picture of the QDs. It is quite helpful to analyze the numerical results for the linear conductance spectrum, as follows. We then introduce the electron creation(annihilation) operators corresponding to the molecular orbits, i.e., $f_{m\sigma}^\dagger$ ($f_{m\sigma}$). By the diagonalization of the single-particle Hamiltonian of the QDs, we find the relation between the molecular and atomic pictures (here each QD is regarded as an “atom”). This is expressed as $[\mathbf{f}_\sigma^\dagger] = [\boldsymbol{\eta}][\mathbf{d}_\sigma^\dagger]$. The $N \times N$ transfer matrix $[\boldsymbol{\eta}]$ consists of the eigenvectors of the QD Hamiltonian. In the molecular orbital picture, the single-particle Hamiltonian takes the form: $\mathcal{H} = \sum_{k\sigma\alpha \in L,R} \varepsilon_{\alpha k} c_{\alpha k\sigma}^\dagger c_{\alpha k\sigma} + \sum_{m=1,\sigma} e_m f_{m\sigma}^\dagger f_{m\sigma} +$

$\sum_{\alpha k\sigma} v_{\alpha m} f_{m\sigma}^\dagger c_{\alpha k\sigma} + \text{h.c.}$, in which e_m is the eigenenergy of the QDs; $v_{\alpha m} = \eta_{mj} V_{\alpha j} + \eta_{m,j+1} V_{\alpha j+1}$, denotes the coupling between the eigenstate e_m and $|k, \sigma\rangle$ in lead- α . In the molecular orbital picture the retarded Green function is defined as $\mathcal{G}_{mm',\sigma}^r = \langle\langle f_{m\sigma} | f_{m'\sigma}^\dagger \rangle\rangle$. We can define $\gamma_{mn}^\alpha = 2\pi v_{\alpha m} v_{\alpha n}^* \rho_\alpha(\omega)$ which denotes the coupling coefficient between the eigenstate e_m and the leads, and their diagonal elements are given by

$$\gamma_{mm}^\alpha = |\eta_{mj} \sqrt{\Gamma_{jj}^\alpha} + \eta_{m,j+1} \sqrt{\Gamma_{j+1,j+1}^\alpha} e^{i\phi_\alpha}|^2. \quad (8)$$

It is known that for the N -QD structure with $t_m = t_0$ and $\varepsilon_m = \varepsilon_0$, the eigenenergies are given by $e_m = \varepsilon_0 - 2t_0 \cos(\frac{m\pi}{N+1})$ and the $[\boldsymbol{\eta}]$ matrix is expressed as

$$[\boldsymbol{\eta}] = \sqrt{\frac{2}{N+1}} \begin{bmatrix} \sin \frac{N^2\pi}{N+1} & \sin \frac{N(N-1)\pi}{N+1} & \cdots & \sin \frac{N\pi}{N+1} \\ \sin \frac{N(N-1)\pi}{N+1} & \sin \frac{(N-1)^2\pi}{N+1} & \cdots & \sin \frac{(N-1)\pi}{N+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sin \frac{N\pi}{N+1} & \sin \frac{(N-1)\pi}{N+1} & \cdots & \sin \frac{\pi}{N+1} \end{bmatrix}.$$

Hence we can see that only in the case of $\phi_\alpha = n\pi$, it is possible for γ^α to be equal to zero so that the corresponding eigenstate decouples from lead- α .

III. NUMERICAL RESULTS AND DISCUSSIONS

With the formulation developed in the previous section, we can perform the numerical calculation to investigate the linear conductance spectrum of this variational parallel double QD structure, namely, to calculate the conductance as a function of the incident electron energy. Prior to the calculation, we need introduce a parameter t_0 as the units of energy.

We choose the parameter values $t_m = \Gamma = t_0$ for the QDs to carry out the numerical calculation. And ε_0 , the QD level, can be shift with respect to the Fermi level by adjusting gate voltage experimentally. Figure 2 shows the linear conductance spectra (\mathcal{G} versus ε_0) for several structures with the QD number $N = 2$ to 4. It is obvious that the $N = 2$ structure just corresponds to the parallel double QDs with interdot coupling, which has been mentioned in previous works. Its conductance spectrum presents a Breit-Wigner lineshape in the absence of magnetic flux, as shown in Fig.2(a). Such a result can be analyzed in the molecular orbital representation. Here the $[\boldsymbol{\eta}]$ matrix, denoting the relation between the molecular and

‘atomic’ representations, takes a form as $[\boldsymbol{\eta}] = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$. Then with the help of Eq. (8) one can find that

here the bonding state completely decouples from the leads and only the antibonding state couples to the leads, which leads to the appearance of the Breit-Wigner lineshape in the conductance spectrum. Besides, introducing the magnetic flux with $\phi_L = \phi_R = \phi = \pi$ (Hereafter we employ ϕ to denote the magnetic flux through each subring for the $\phi_L = \phi_R$ case) can change the decoupling state, as exhibited by the dashed line in Fig.2(a). In such a case, only the bonding state couples to the leads and the conductance profile also shows a Breit-Wigner lineshape.

In Fig.2(b) the conductance curves as a function of gate voltage are shown for the 3-QD structure. Obviously, there exist three conductance peaks in the conductance profiles and no decoupling quantum state appears. We can clarify this result by calculating $\gamma_{mm}^\alpha = \Gamma |\eta_{m1} + \eta_{m2} e^{i\phi}|^2$, and it is obvious that γ_{mm}^α is

impossible to be equal to zero in such a structure despite the adjustment of magnetic flux. Thus one can not find the decoupling eigenstates, the state-lead coupling may be relatively weak, though. Just as shown in Fig.2(b), in the absence of magnetic flux the distinct difference of the couplings between the quantum states and leads offer the ‘more’ and ‘less’ resonant channels for the quantum interference. Then the Fano effect occurs and the conductance profile presents an asymmetry lineshape. In addition, the Fano lineshape in the conductance spectrum is reversed by tuning the magnetic flux to $\phi = \pi$, due to the modulation of magnetic flux on γ_{mm}^α .

When the QD number increases to $N = 4$, there will be two configurations corresponding to this structure, i.e., the cases of $j = 1$ and $j = 2$. As a consequence, the conductance spectra of the two structures remarkably differ from each other. With respect to the configuration of $j = 1$, the electron transport properties presented by the conductance spectra are similar to those in the case of the 3-QD structure, as shown in Fig.2(c), and there is also no occurrence of decoupling states. However, as for the case of $j = 2$, it is clear that in the absence of magnetic flux, there are two conductance peaks in the conductance spectrum, which means that the decoupling phenomenon comes into being. Alternatively, in the case of $\phi = \pi$, there also exist two peaks in the conductance profile. But the conductance peaks in the two cases of $\phi = 0$ and π do not coincide with one another. We can therefore find that in this structure, when $\phi = n\pi$ decoupling phenomena will come up and the adjustment of magnetic flux can effectively change the appearance of decoupling states. By a calculation and focusing on the conductance spectra, we can anticipate that in the case of $\phi = 2n\pi$, the odd (first and third) eigenstates decouple from the leads; In contrast, the even (second and fourth) eigenstates of the QDs will decouple from the leads if $\phi = (2n - 1)\pi$. In addition, as shown in Fig.2(d), the conductance encounters its zero when the level of QDs is the same as the Fermi level of the system. Furthermore, the conductance zero, usually called antiresonance, is irrelevant to the tuning of magnetic flux, even though the changes of couplings between the eigenstates and leads.

In order to obtain a clear physics picture about decoupling, we analyze this problem in the molecular orbital representation. By solving the $[\eta]$ matrix and using the relation $\gamma_{mm}^\alpha = \Gamma|\eta_{m2} + \eta_{m3}e^{i\phi}|^2$, it is easy to find in the case of zero magnetic flux, γ_{11}^α and γ_{33}^α is always equal to zero, which brings out the completely decoupling of the odd eigenstates from the leads. Opposite to this case, when $\phi = \pi$ the values of γ_{22}^α and γ_{44}^α are fixed at zero. And such a result leads to the even eigenstates to decouple from the leads. However, the underlying physics responsible for antiresonance is desirable to clarify. We then analyze the electron transmission in the molecular representation. We take the case of $\phi = 0$ as an example, where only two quantum states e_2 and e_4 couple to the leads due to decoupling. Accordingly, e_2 and e_4 might be called as well the bonding and antibonding states. As is known, the molecular orbits of coupled double QD structures, e.g., the well-known T-shaped QDs, are regarded as the bonding and antibonding states. Therefore, by employing the representation transformation $[\mathbf{a}_\sigma^\dagger] = [\beta][\mathbf{f}_\sigma^\dagger]$, such a configuration can be changed into the T-shaped double-QD system of the Hamiltonian $H = \sum_{k\sigma\alpha \in L,R} \varepsilon_{\alpha k} c_{\alpha k\sigma}^\dagger c_{\alpha k\sigma} + \sum_{\sigma,n=1}^2 \epsilon_n a_{n\sigma}^\dagger a_{n\sigma} + t a_{2\sigma}^\dagger a_{1\sigma} + \sum_{\alpha k\sigma} w_{\alpha 1} a_{1\sigma}^\dagger c_{\alpha k\sigma} + h.c..$

By a further derivation, the relations between the structure parameters of the two QD configurations can be obtained with $\epsilon_1 = \varepsilon_0 + t_0$, $\epsilon_2 = \varepsilon_0$, $t = t_0$, and $w_{\alpha 1} = V_{\alpha 1}$ respectively. Accordingly, we have $\gamma_{22}^\alpha = \Gamma|\beta_{11}|^2$,

and $\gamma_{44}^\alpha = \Gamma|\beta_{21}|^2$ with $[\beta] = \frac{1}{\sqrt{2\sqrt{5}}} \begin{bmatrix} -\sqrt{\sqrt{5}-1} & \frac{2}{\sqrt{\sqrt{5}-1}} \\ \sqrt{\sqrt{5}+1} & \frac{2}{\sqrt{\sqrt{5}+1}} \end{bmatrix}$. The 4-QD structure is then transformed into

the T-shaped double QDs with ε_0 being the level of dangling QD. Just as discussed in the previous works[29], in the T-shaped QDs antiresonance always occurs when the dangling QD level is aligned with the Fermi level of the system. With the help of such an analysis, one can then understand that in this 4-QD system, the antiresonant point in the conductance spectrum is consistent with $\varepsilon_0 = 0$.

The occurrence of antiresonance in the T-shaped QDs can be interpreted as the quantum interference between two kinds of transmission paths. We demonstrate this issue by rewriting the Hamiltonian of the T-shaped double QD structure as $H = \mathcal{H}_0 + \mathcal{H}_t$ in which

$$\begin{aligned} \mathcal{H}_0 &= \sum_{k\sigma} \xi_{Lk} \alpha_{Lk\sigma}^\dagger \alpha_{Lk\sigma} + \sum_{kk'\sigma} (t_{kk'} \alpha_{Lk\sigma}^\dagger c_{Rk'\sigma} + h.c.) \\ &\quad + \sum_{k\sigma} \varepsilon_{Rk} \alpha_{Rk\sigma}^\dagger c_{Rk\sigma} + \sum_{\sigma} \varepsilon_2 a_{2\sigma}^\dagger a_{2\sigma}, \\ \mathcal{H}_t &= \sum_{k\sigma} \nu_k \alpha_{Lk\sigma}^\dagger a_{2\sigma} + h.c.. \end{aligned} \quad (9)$$

Here the old operators $a_{1\sigma}$ and $c_{Lk\sigma}$ are expanded in terms of this new set: $a_{1\sigma} = \sum_k \nu_k \alpha_{Lk\sigma}$ and $c_{Lk\sigma} = \sum_{k'} \eta_{k,k'} \alpha_{Lk'\sigma}$. Under this new representation, the electron transmission paths are well described: One is an electron transmission path whereby the electron starts from the left lead tunnels directly into the right lead,

the other is a different transmission path from the above one in that the electron must visit the dangling QD as it tunnels through the QD structure. Note that the electron visiting the dangling QD will result in its phase change, and the phase difference between the two kinds of paths gives rise to the destructive quantum interference. In Ref.[34], a detailed discussion is presented.

When paying attention to the $[\eta]$ matrix, one will see that $\eta_{12} = \eta_{43}$, $\eta_{22} = -\eta_{33}$, $\eta_{32} = \eta_{23}$, and $\eta_{42} = -\eta_{13}$ for the 4-QD structure. As a result, such relations give rise to $\gamma_{22}^\alpha|_{\phi=0} = \gamma_{33}^\alpha|_{\phi=\pi}$ and $\gamma_{44}^\alpha|_{\phi=0} = \gamma_{11}^\alpha|_{\phi=\pi}$. So, when $\phi = \pi$ the magnetic flux reverses the lineshape of the conductance spectrum in the case of $\phi = 0$. Based on these properties, we can realize that the quantum interference in this case is similar to that in the case of $\phi = 0$. Therefore, the antiresonant point in the conductance spectra is independent of the adjustment of magnetic flux.

By virtue of Eq.(8) we can find that in the situation of $\phi_\alpha = n\pi$ and $\phi_{\alpha'} \neq n\pi$, some eigenstates of the QDs will decouple from lead- α but they still couple to lead- α' . Figure 3 shows the research on electron transport within the 2-QD and 4-QD $j = 2$ structures. For the 2-QD structure, it is obvious that in the case of $\phi_L = 0$ and $\phi_R = 0.5\pi$ both the bonding and antibonding states couple to lead-R but the bonding state decouples from lead-L. The numerical result is shown in Fig.3(a). Clearly, due to decoupling antiresonance occurs at the point of $\varepsilon_0 = t_0$. This indicates that in such a case antiresonance will come into being when the decoupling state is tuned to be consistent with the Fermi level. As discussed in our previous works, this antiresonance arises from the destructive interference among electron going through two kinds of transmission paths. Then in the case of $\phi_L = 0$ and $\phi_R = \pi$, the bonding state decouples from lead-L with the antibonding state decoupling from lead-R. So in this case there is no channel for the electron tunneling and the conductance is always equal to zero, despite the shift of gate voltage. On the other hand, by fixing $\phi_R = \pi$ and increasing ϕ_L to 0.5π , one will see that the decoupling of antibonding state from lead-R results in the antiresonance at the point of $\varepsilon_0 = -t_0$. With regard to the 4-QD with $j = 2$ structure, the decoupling-induced antiresonance is also remarkable in the case of $\phi_\alpha = n\pi$ and $\phi_{\alpha'} \neq n\pi$. As shown in Fig.3(b) there are two kinds of antiresonant points in such a case: one originates from the quantum interference between the connecting eigenstates, and the other is caused by the decoupling states.

Based on the above analysis, we can expect that in the structure of with $t_m = t_0$ and $\varepsilon_m = \varepsilon_0$, when N is even and $j = \frac{N}{2}$ there must be the appearance of decoupling eigenstates. This expectation can be confirmed because of $\eta_{mj} = -\eta_{m,j+1}$ ($m \in \text{odd}$) and $\eta_{mj} = \eta_{m,j+1}$ ($m \in \text{even}$) for the QDs. The numerical results in Fig.4, describing the conductances of 6-QD and 8-QD structures, can support our conclusion. Besides, with the help of the representation transformation the positions of antiresonance can be clarified by transforming these structure into the T-shaped QD systems.

In Fig.5 the linear conductances of the semi-infinite and infinite QD chains are presented as a function of gate voltage. As shown in Fig.5(a)-(b), for the case of semi-infinite QD chains with $j = 1$ and 2, there is no conductance peaks consistent with any eigenlevel since the eigenstates of the QDs become a continuum in such a case. It should be pointed out that although some eigenstates decouple from the leads, it can not affect the electron transport since the electron transmission paths can not be differentiated in the case of continuum. Thus no antiresonance appears in the conductance spectra. With this point of view, it is easy to understand that for the infinite QD chain, there is also no antiresonance in the conductance profile, as shown in Fig.5(c). However, when investigating the influence of the difference between ϕ_L and ϕ_R on the electron transport, we find that in the situation of $\phi_L = 0$ and $\phi_R = 0.5\pi$ the conductance of the infinite QD chain encounters its zero at the down side of energy band, as shown in Fig.6(b). Such a result can be explained as follows. The coupling of a semi-infinite QDs to QD- j indeed brings out an additional self-energy to its level, which can be written out explicitly as $\Sigma = \frac{1}{2}(-\varepsilon_0 - i\sqrt{4t_0^2 - \varepsilon_0^2})$, which renormalizes the level of QD- j ε_0 to $\varepsilon_0 + \Sigma$. At the upper side of the energy band, the renormalized QD level becomes $\frac{\varepsilon_0}{2}$ because of $\Sigma = -\frac{\varepsilon_0}{2}$ here. Similarly, the level of QD- $(j+1)$ is equal to $\frac{\varepsilon_0}{2}$. Therefore, at this point the structure is transformed into the double-dot configuration. Based on our discussion on the quantum interference of the double-dot system, we know that when the bonding state $e_1 = \frac{\varepsilon_0}{2} - t_0$ is aligned with the Fermi level, the linear conductance presents antiresonance. With these knowledge, one can clarify the occurrence of conductance zero in such a case. Alternatively, a similar reason gives rise to antiresonance at the up side of the energy band for the case of $\phi_L = 0.5\pi$ and $\phi_R = \pi$. In addition, it is apparent that in the case of $\phi_L = 0$ and $\phi_R = \pi$ the conductance is always fixed at zero. This result can be readily explained that in such a situation any eigenstate coupled to lead- α is inevitable to decouple from lead- α' , though the eigenstates of the QDs is continuum. Thus, there is still no channels for the electron transport.

Before concluding, we have to make a remark regarding the many-body effect which we have by far ignored. As is known, the many-body effect is an important origin for the peculiar transport properties in QDs. Usually, the many-body effect is incorporated by considering only the intradot Coulomb repulsion, i.e., the Hubbard term. If the Hubbard interaction is not very strong, we can truncate the equations of motion of the Green functions to the second order. By a straightforward derivation, we find that in such an

approximation, the Green function is redefined as

$$g_{m\sigma}(z) = \left[\frac{z - \varepsilon_m}{1 + \frac{U_m \langle n_{m\bar{\sigma}} \rangle}{z - \varepsilon_m - U_m}} + i\Gamma_{mm} \right]^{-1}. \quad (10)$$

It can be expected that the conductance spectrum will be split into two groups[35, 36, 37], and in each group the electron transport properties in the noninteracting case will remain.

IV. SUMMARY

With the help of nonequilibrium Green function technique, the electron transport through a QD chain is theoretically studied. In such a system both the leads couple to the two neighboring QDs of the chain. It has been found that only for the even-numbered QD structure with the same-number QDs coupling to the connecting QDs, some eigenstates of such coupled QDs decouple from the leads. To be concrete, in the absence of magnetic flux the odd eigenstates of the QDs decouple from the leads, whereas its even eigenstates decouple from the leads when an appropriate magnetic flux is introduced. In addition, the antiresonance in the conductance spectra is irrelevant to the tuning of magnetic flux. By means of the representation transformation, such phenomena were analyzed in detail. By adjusting the magnetic fluxes through each subring, we found that some eigenstates of these QDs decoupled from one lead but still coupled to the other, which causes the occurrence of new antiresonance. These results vanish for the case of the infinite QD chain.

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FIG. 1: Schematic of the QD chain with two neighboring QDs coupled to both leads. Two magnetic fluxes Φ_L and Φ_R thread the subrings in the structure.

FIG. 2: The linear conductance spectra of N-QD chains with $N = 2$ to 4. The structure parameters take the values as $\Gamma = t_m = t_0$, with t_0 being the unit of energy.

FIG. 3: The calculated conductance spectra of the 2-QD and 4-QD structures by the presence of $\phi_\alpha = n\pi$ and $\phi_{\alpha'} \neq n\pi$.

FIG. 4: (a) The conductances of 6-QD system with $j = 3$. (b) The conductances of 8-QD structure in the case of $j = 4$.

FIG. 5: The conductances of the semi-infinite and infinite QD chains.

FIG. 6: The conductances of the semi-infinite and infinite QD chains in the presence of $\phi_\alpha = n\pi$ and $\phi_{\alpha'} \neq n\pi$.











